**Spin s = 1 in B field**

(believe γ is signed quantity in these notes, i.e., negative for an electron) We’ll recall,



**Example. Spin in NMR field**

Going to try the time-dependent NMR field again. But this time, we’ll not go to the interaction picture. It seems to be good for perturbation theory, but maybe not the best if want to try for an exact solution. So our H is:



We’ll ultimately set the fields to constants. Distributing the dot product,



Projecting onto the spin basis we get:



Then calling B(t) = Bx(t) + iBy(t), we have:



Keeping the fields time-dependent,



Let’s look for eigenvalues,



Eigenvectors are:



Only two equations are linearly independent. Let’s use the top/bottom.



Choose c2 = √2. Then,



Normalization is:



**λ = 0 …**

So we get:



**λ = √(|B|2 + Bz2) …**

So we get (I’m going to change the normalization by factor of -1 to make it positive)



**λ = -√(|B|2 + Bz2) …**

So we get (noting normalization is invariant w/r to sign of λ):



So we can write V as:



We can also write U as,



So we can say,



Then we introduce the definition,



to write,



So this is an effectively time-independent problem now. With effective H,



Of course U0ΛU0ⴕ is the time-independent Hamiltonian, or, well, the Hamiltonian at time t = 0. So Heff is:



Fortunately for us, we don’t have to re-diagonalize Heff. Well, we do, but it’s easy. We can just take our prior constant field result, and make the replacement Bz -> Bz + ω/gγ. So we can write,



where,



Given this, we can continue with our solution,



Define,



and we have:



And the solution is:



So….



where,



Let’s go ahead and work this out, taking advantage of our earlier work on the time-independent case. So, it’s probably easiest to look at (t). We just have to make the Bz -> Bz + ω/gγ replacement. So doing this, and changing notation B -> BT, BT2 + Bz2 -> B2, and moreover,



We have:



and,



And,



And now we use,



to come to,







That was fun. Should’ve done the s=1/2 case this way. Can NMR for any spin quantum number (or really, j quantum number) be done this way? I suspect so. Just a lot of algebra then. Again,



**Transitions out of |-1>**

Let’s consider transitions starting from state |-1>.



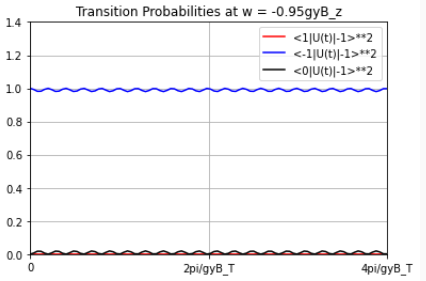
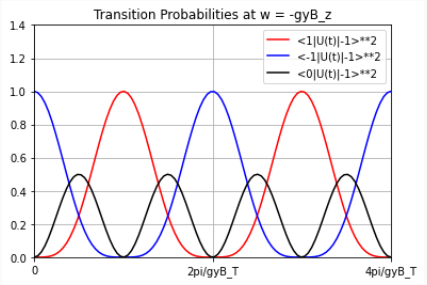
And let’s take ω = -gγBz. Then Bz(ω) = 0, and B(ω) = BT. And we have:



The transition probabilities would be:



Here’s a rough plot of the forms of the transition probabilities as a function of time. Observe that we have to be very close to resonance to have a reasonable chance of exciting a transtition. Even 95% resonance doesn’t really move the needle.

Do these add up to 1?



That’s good. So interestingly, transitions from |1> to |0> are initially more likely than |1> to |-1>. This makes sense because |1> to |0> is direct, but |1> to |-1> is indirect (see perturbation theory). But also note that |1> to |-1> eventually has the largest amplitude. So the two-step ‘photon’ absorption process is more strongly encouraged here.

Note that V(t) doesn’t couple |1> to |-1> (and vice versa). And this seems to be why there is no resonance at ω = 2gγBz, as we would otherwise directly see from 1st order Perturbation Theory. So it looks like transitions from |1> to |-1> can’t happen directly via absorption of a single photon. Rather, it happens in a two-step process whereby a photon is absorbed to effect the transtion |1> to |0> and then another photon is absorbed to accomplish |0> to |-1>.

**Transitions out of |0>**

What about the amplitudes for starting off in state |0>?



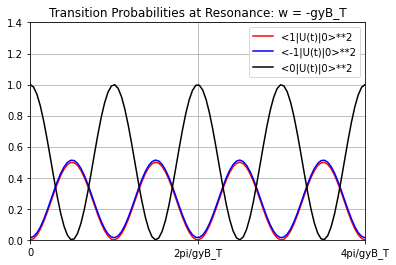
and when ω = -gγBz,



And probabilities,



These obviously add up to 1. And here’s a rough plot of the general form.



(that should say w = -gγBz) And here, even in resonance, neither state |-1> or |1> gets completely populated, which kind of makes sense since they are both equidistant from |0>, energy-wise. Cool.

**Small BT Limit**

Let’s compare to RSPT. We’ll expand these results to second order in BT, as applicable,



We can see a factor of t showing up, which would make this blow up in the large t limit. Next,



and finally,



We’ll note that in the limit ω -> -gγBz, the last two terms cancel, and we’re left with:



This result reproduces first order Time Dependent RSPT, which we’ll cover in later files. If we continue to take the limit, we’ll get:



And this indicates that, at this level of perturbation theory, the transition amplitude, and probabality consequently, increases with time. Eventually, it would surpass 1, and so we’d think that the probability of making a transition to this state is inevitable for long times. But in fact it is not. If we look at at the exact result, we see that the amplitude for the transition |-1> to |0> maxes out at 1/√2. And in fact, it is the transition from |-1> to |1> which goes to 1, periodically, even though it was 0, to first order in the perturbation. So this tells us to take the results of perturbation theory, especially time-dependent perturbation theory, with a grain of salt. Another related point: from perturbation theory, we’d think that we need an oscillating field to with frequency ω = 2gγBz to pump a state from |-1> to |1> with probability 1. But I guess this is only what the first order result tells us. In our present case, the first order term is zero because the perturbation, BT, doesn’t connect |-1> and |1> at first order. It does at second order though, and as we can see, if we wait long enough, then the ω = gγBz frequency suffices.